Asymptotic Combinatorics with Application to Mathematical Physics
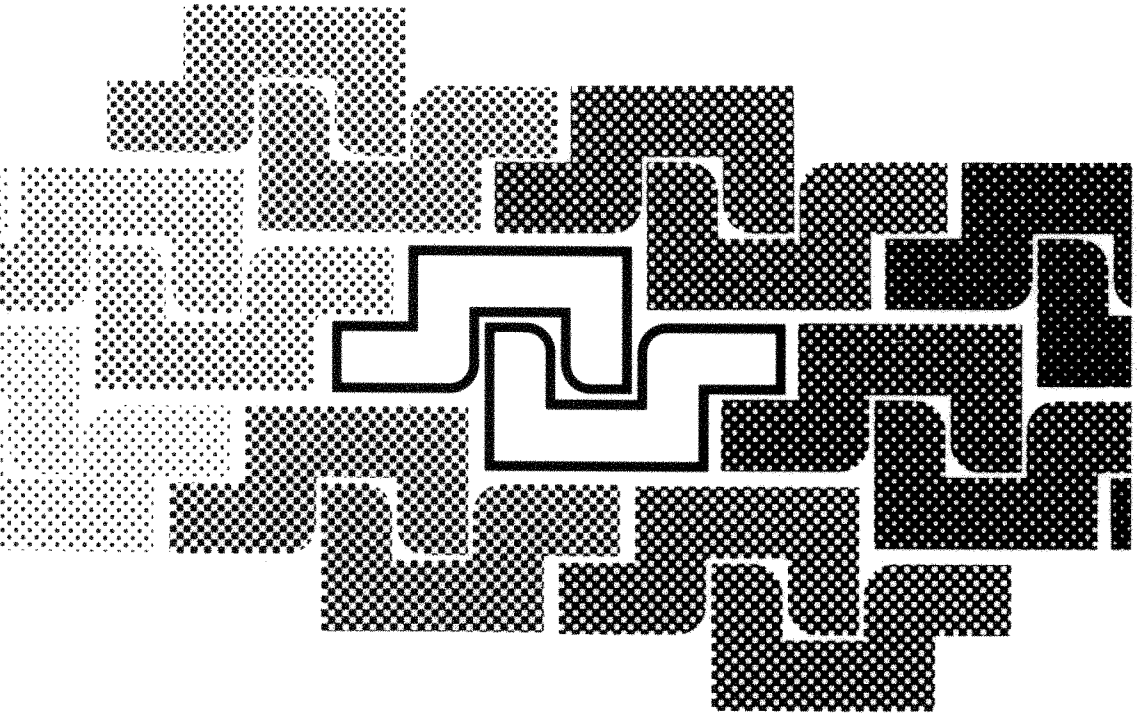
Edited by

Vadim Malyshev and Anatoly Vershik

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Asymptotic Combinatorics with Application to Mathematical Physics

edited by

Vadim Malyshev
I.N.R.I.A.,
Le Chesnay, France

and

Anatoly Vershik
Steklov Mathematical Institute of Russian Academy of Sciences,
St. Petersburg, Russia

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Contents

PREFACE

This volume presents some courses and series of lectures which were organized in the framework of NATO Advanced Study Institute under the title

"Asymptotic Combinatorics with Application to Mathematical Physics"

at Euler International Mathematical Institute at St. Petersburg, July 2001. At the same time it was European Summer School as a part of the activity of European Mathematical Society. The idea of organizers was to invite the best specialists in this area to give a series of lectures for young mathematicians as well as for specialisits who are interested in recent progress in wide part of mathematics including asymptotic methods in mathematical physics, combinatorics, representation theory and some applications. One of the main points in the organization of the School was to invite both mathematicians and physicists in order to emphsize deep connections and interrelation between various approaches to the common subjects. This volume includes some courses and seminar talks which are more concerned to mathematical physics.

The areas of mathematics and mathematical physics which were the subject of the meeting have been studied very intensively last years. This intensive investigations brought about a series of striking results which could be considered together as a new higher level of understanding of related problems: theory of integrable systems, Riemann–Hilbert problem, asymptotic representation theory, spectrum of random matrices, combinatorics of Young diagrams and permutations and even some aspects of quantum field theory. The initial idea to get leading specialists in all these areas together in order to give short courses of lectures on these subjects and to attract attention to recent progress, and especially to attract young mathematicians to those areas, had met an active reaction of the specialists, and we hope it ended up being a fruitful contact between asymptotic combinatorics and mathematical physics. The list of lecturers included the pioneers in these areas — E. Brezin, P. Deift, L. Faddeev, V. Kazakov, S. Novikov, et al).

The methods of theory of integrable systems, matrix problems and theory of Riemann–Hilbert problem together with asymptotic combinatorics and representation theory give very powerful tools for solving numerous old problems: distribution of the fluctuations of the eighenvalues of random matrices, counting the number of coverings of algebraic curves, universality of the distribution of the spacing and other statistical characteristics of Young diagrams, etc. It make sense to mention the lectures and courses about spectacular new results in mathematical physics (P. Deift, L. Pastur), matrix problems (E. Brezin, V. Kazakov), applications of combinatorics (A. Borodin, R. Kenyon), representation theory (A. Vershik, G. Olshansky) algebraic geometry (A. Okounkov), probability theory of maps (V. Malyshev) as well as other distinguished lectures (L. Faddeev, S. Novikov, I. Krichever, V. Korenpin, J. Jacobsen et al)

We do not want to comment each paper in this volume but nevertheless emphasize that combinatorial and probabilistic methods started to play an important role in mathematical physics. Random matrix models, combinatorics of maps, and Young diagrams as well as traditional methods of generating functions and related part of analysis become more and more popular and powerful in applications to mathematical physics. Simultaneously these needs give a serious additional impulse for the pure mathematical theory of these objects. We can watch also a fast developing of nontraditional probabilistic theories based on new achievements. All this shows the fruitful interrelation between these new parts of mathematics and physics.

The program of the school was even overfull with additional talks (besides lectures). The Round table on the current problems and some special session with the questions and answers for young people and short discussions about their studies were organized during the school. We decided to include in this volume some papers of authors who unfortunately could not attend to the meeting but have sent us their papers (V. Maslov, D. Aldous and J. Pitman etc).

The NATO ASI — EMS School was organized by International Euler Mathematical Institute, St. Petersburg branch of Mathematical Institute of Russian Academy of Sciences (POMI RAN) and St. Petersburg Mathematical Society. The preparation of the School started at 1999; application to NATO was sent in July 2000 when the School was already in preparation as European Summer School in the framework of Summer schools of the European Mathematical Society. The support which came from the NATO Scientific Affairs Division was extremely important. Without this support the School could not be so representative and successful. Below we put the more details on the school.

V. A. MALYSHEV
A. M. VERSHIK

Asymptotic combinatorics with application to mathematical physics

PROGRAM

July 8–20

July 8, Sunday (Euler International Mathematical Institute, Pesochnaya
     nab., 10)
15.00–20.00 REGISTRATION

1st day, July 9, Monday (Steklov Mathematical Institute, Fontanka, 27)
08.30–09.45 REGISTRATION
10.00–10.30 Opening Session
10.30–11.30 *Brezin E.* An introduction to matrix models – 1
12.00–13.00 *Vershik A.* Introduction to asymptotic theory of representations – 1
15.00–15.50 *Korepin V.* Quantum spin chains and Riemann Zeta function with
     odd arguments
16.10–17.00 *Bozejko M.* Positive definite functions on Coxeter groups and
     second quantization of Yang–Baxter type

2nd day, July 10, Tuesday (Euler International Mathematical Institute,
     Pesochnaya nab., 10)
9.30–10.30 *Ol'shanski G.* Harmonic analysis on big groups, and determinantal
     point processes – 1
10.40–11.40 *Brezin E.* An introduction to matrix models – 2
12.10–13.10 *Malyshev V.* Combinatorics and probability for maps on two
     dimensional surfaces
15.00–15.50 *Hora A.* An algebraic and combinatorial approach to central limit
     theorems related to discrete Laplacians
16.00–16.50 *Nazarov M.* On the Frobenius rank of a skew Young diagram
17.20–18.10 *Kenyon R.* Hyperbolic geometry and the low-temperature
     expansion of the Wulff shape in the $3D$ Ising model
19.40 *Brezin E.* Informal discussion on matrix models

3rd day, July 11, Wednesday
9.30–10.30 *Okounkov A.* Combinatorics and moduli spaces of curves – 1
10.40–11.40 *Brezin E.* An introduction to matrix models – 3
12.10–13.10 *Biane Ph.* Asymptotics of representations of symmetric groups,
     random matrices and free cumulants
14.00–17.00 Excursion over St. Petersburg
19.00 Mariinski Theatre

4th day, July 12, Thursday
9.30–10.30 *Deift P.* Random matrix theory and combinatorics: a
     Riemann–Hilbert approach – 1
10.40–11.40 *Okounkov A.* Combinatorics and moduli spaces of curves – 2

12.10–13.10 **Kazakov V.** Matrix quantum mechanics and statistical physics on planar graphs – 1
15.00–15.50 **Faddeev L.** 3-dimensional solitons and knots
16.30–18.10 **Krichever I.** τ-functions of conformal maps
19.30 Boat trip

5th day, July 13, Friday
9.30–10.30 **Deift P.** Random matrix theory and combinatorics: a Riemann–Hilbert approach – 2
10.40–11.40 **Okounkov A.** Combinatorics and moduli spaces of curves – 3
12.10–13.10 **Ol'shanski G.** Harmonic analysis on big groups, and determinantal point processes – 2
15.00–15.50 **Kazakov V.** Matrix quantum mechanics and statistical physics on planar graphs – 2
16.00–16.50 Round table on combinatorics of the configurations and limit shapes
17.20–18.10 **Liskovets V.** Some asymptotic distribution patterns for planar maps

July 14 Excursions, Museums, etc.

July 15 Excursion to Petergof 10:00 (from Euler Institute)

6th day, July 16, Monday
9.30–10.30 **Deift P.** Random matrix theory and combinatorics: a Riemann–Hilbert approach – 3
10.40–11.30 **Kazakov V.** Combinatorics of planar graphs in matrix quantum mechanics – 3
12.10–13.10 **Borodin A.** Asymptotic representation theory and Riemann–Hilbert problem – 1
15.00–15.50 **Novikov S.** On the weakly nonlocal Poisson and Symplectic Structures
16.00–16.50 **Spiridonov V.** Special functions of hypergeometric type associated with elliptic beta integrals
17.20–18.10 **Missarov M.** Exactly solvable renormalization group model

7th day, July 17, Tuesday
9.30–10.30 **Deift P.** Random matrix theory and combinatorics: a Riemann–Hilbert approach – 4
10.40–11.40 **Borodin A.** Asymptotic representation theory and Riemann–Hilbert problem – 2
12.10–13.10 **Okounkov A.** Combinatorics and moduli spaces of curves – 4
15.00–15.50 **Speicher R.** Free probability and Random matrices
16.00–16.50 **Korotkin D.** Riemann–Hilbert problems related to branched coverings of GP^1, τ-function and Liouville action
17.20–18.10 **Litvinov G.** Representation theory in Idempotent (asymptotic) Mathematics

8th day, July 18, Wednesday

9.30–10.30 *Smirnov S.* Critical percolation is conformally invariant – 1

10.40–11.40 *Pastur L.* Eigenvalue distribution of unitary invariant ensembles of random matrices of large order – 1

12.10–13.10 *Pevzner M.* On tensor products and Beresin kernels

15.00–17.00 *Memorial session devoted to Sergei Kerov and Anatoly Izergin*

9th day, July 19, Thursday

9.30–10.30 *Speicher R.* Free probability and Random matrices – 2

10.40–11.40 *Bozejko M.* White noise associated to the characters of the infinite symmetric group — Hopf–Kerov deformation

12.10–13.00 *Jacobsen J. L.* Enumerating coloured tangles

13.05–13.35 *Sniady P.* Random matrices and free probability

15.00–15.30 *Mlotkowski W.* Λ-free probability

15.30–16.00 *Petrogradskij V.* Asymptotical theory of infinite dimensional Lie algebras

16.30–17.00 *Kuznetsov V.* On explicit formulae for special Macdonald polynomials

17.05–17.35 *Dubrovskiy S.* Moduli space of symmetric connections

17.40–18.10 *Stukopin V.* Representations of Yangians of Lie Superalgebras $A(m,n)$ type

10th day, July 20, Friday

9.30–10.30 *Pastur L.* Eigenvalue distribution of unitary invariant ensembles of random matrices of large order – 2

10.40–11.10 *Vershik A.* Introduction to asymptotic theory of representations – 2

11.15–11.45 *Yambartsev A.* Two-dimensional Lorentzian Models

12.15–13.00 *Round table*: Problems of the theory of integrable operators and determinant processes

13.00–13.30 *Closing the school*

List of participants

Name	Affiliation	email
Ahmedov Ruslan	Moscow State University, Russia	ahmedov@mccme.ru
Alexandrov Sergej	SPhT, France	alexand@spht.saclay.cea.fr
Aptekarev Alexander	Keldysh Institute, Russia	aptekaa@spp.keldysh.ru
Balog Jozsef	University of Memphis, USA	balogj@msci.memphis.edu
Batchourine Pavel	Moscow State University, Russia	bachurin@ipac.ac.ru
Belogrudov Alexander	Inst. of Mathematics, Ufa Science Centre of RAS; Russia	belan@anrb.ru
Beliaev Dmitri	KTH, Sweden	beliaev@math.kth.se
Biane Philippe	CNRS, ENS, DMA, France	Philippe.Biane@ens.fr
Bloznelis Mindaugas	Vilnius University, Lithuania	mblozn@ieva.maf.vu.lt
Borodin Alexey	University of Pennsylvania, USA	borodine@math.upenn.edu
Bozejko Marek	Instytut Matematyczny, Wroclaw University, Poland	bozejko@math.uni.wroc.pl
Brezin Edouard	Ecole Normale Superiore, France	brezin@corto.lpt.ens.fr
Calkin Neil	Clemson University; USA	calkin@math.clemson.edu
Carlet Guido	SISSA, Italy	carlet@sissa.it
Chebalov Sergej	UIUC, USA	chebalov@math.uiuc.edu
Chou Ching-Sung	Dept of Math, National Central University, Taiwan, R.O.C.	chou@math.ncu.edu.tw
Cislo Jerzy	Univ of Wroclaw, Institute of Theoretical Physics, Poland	cislo@ift.uni.wroc.pl
Cutler Johnatan	University of Memphis, USA	cutlerj@msci.memphis.edu
Deift Percy	Courant Institute and University of Pennsylvania, USA	deift@math1.cims.nyu.edu
Dobrynin Sergei	St. Petersburg State University, Russia	
Dubrovskiy Stanislav	Northeastern University, USA	dubr@research.neu.edu
Duzhin Sergei	Steklov Institute of Mathematics at St. Petersburg (POMI), Russia	duzhin@pdmi.ras.ru
Erschler Anna	Steklov Institute of Mathematics at St. Petersburg (POMI), Russia	erschler@pdmi.ras.ru
Faddeev Ludvig	Steklov Institute of Mathematics at St. Petersburg (POMI), Russia	faddeev@pdmi.ras.ru
Fedotov Sergej	UMIST, UK	sergei.fedotov@umist.ac.uk
Galkin Sergej	Moscow Independent University, Russia	
Gioev Dimitri	KTH, Sweden	
Gorbulski Alexander	St. Petersburg State University, Russia	alexander@MG4812.spb.edu
Gorbulski Mikhail	St. Petersburg State University, Russia	mike@MG4812.spb.edu
Gordin Mikhail	Steklov Institute of Mathematics at St. Petersburg (POMI), Russia	gordin@pdmi.ras.ru
Grebenkov Denis	Ecole Normale Superiore, France	dg@pmc.polytechnique.fr
Gurin Alexey	ILT NAF Ukraina, Ukraine	gurin@ilt.kharkov.ua
Hora Akihito	Okayama University, Japan	hora@ems.okayama-u.ac.jp
Igonine Serguei	University of Twente, the Netherlands	igonin@mccme.ru
Jacobsen Jesper Lykke	Lab de Physique Theorique et Modeles Statistiques, France	jacobsen@ipno.in2p3.fr
Kalita Eugen	Inst. of Applied Mathematics and Mechanics NAS Ukraine	kalita@iamm.ac.donetsk.ua
Kazakov Vladimir	Ecole Normale Superiore, France	Vladimir.Kazakov@lpt.ens.fr
Kenyon Richard	CNRS UMR 8628, France	kenyon@topo.math.u-psud.fr
Khimshiashvili Giorgi	A.Razmadze Mathematical Institute, Georgia	khimsh@rmi.acnet.ge

Name	Affiliation	email
Kirillov Alexander	SUNY at Stony Brook, USA	kirillov@math.sunysb.edu
Kokhas Konstantin	St. Petersburg State University, Russia	kostik@KK1437.spb.edu
Korepin Vladimir	YITP, State University of New York at Stony Brook, USA	korepin@insti.physics.sunysb.edu
Krichever Igor		
Krikun Maxim	Lab of Large Random Systems, Moscow State Univ, Russia	
Kuznetcov Vadim	University of Leeds; UK	vadim@amsta.leeds.ac.uk
Larson Craig	University of Houston; USA	clarson@math.uh.edu
Liskovets Valerij	Institute of Mathematics of NAS of Belarus	liskov@im.bas-net.by
Litvinov Grigori	International Sophus Lie Centre, Russia	islc@dol.ru
Lodkin Andrei	St. Petersburg State University, Russia	lodkin@pdmi.ras.ru
Lucena Brian	Brown University: Division of Applied Math, USA	lucena@cfm.brown.edu
Maida Mylene	ENS, France	maida@clipper.ens.fr
Malyshev Vadim	I.N.R.I.A., France	Vadim.Malyshev@inria.fr
Marciniak Macin	Institute of Mathematics, Gdansk University, Poland	matmm@univ.gda.pl
Mattera Massimiliano	Universite Paris Sud, Laboratoire de mathematiques France	Massimiliano.Mattera@math.u-psud.fr
Matveev Vladimir	Steklov Institute of Mathematics at St. Petersburg (POMI), Russia	matveev@u-bourgogne.fr
Missarov Moukadas	Kazan State University, Department of Applied Mathematics Laboratory of Probability Theory; Russia	Moukadas.Missarov@ksu.ru
Mlotkowski Wojcieh	Mathematical Institute, The University of Wroclaw, Poland	mlotkow@math.uni.wroc.pl
Mnev Nikolai	Steklov Institute of Mathematics at St. Petersburg (POMI), Russia	mnev@pdmi.ras.ru
Momar Dieng	University of California, Davis; USA	momar@math.ucdavis.edu
Nazarov Maxim	Department of Mathematics, University of York, UK	mln1@york.ac.uk
Nikitin Pavel	St. Petersburg State University, Russia	pasha@AN4760.spb.edu
Novikov Sergej	Landau Institute RAN, Moscow, Russia	
Okounkov Andrej	Univ of California at Berkeley, Dept of Mathematics, USA	okounkov@math.berkeley.edu
Olshanlki Grigori	Institute for Information Transmission Problems, Russia	olsh@online.ru
Panov Dmitri	Ecole Polytchnique, CMAT, France	panov@ihes.fr
Pastur Leonid	CPT, France/FTINT NANU, Ukraine; France	pastur@cpt.univ=mrs.fr
Petrogradskij Viktor	Ulyanovsk State University, Russia	vmp@mmf.ulsu.ru
Pevzner Micha	ULB; Belgium	mpevzner@ulb.ac.be
Romanovski Yuri	St. Petersburg State University, Russia	
Rider Brian	Dept of E.E., The Technion, Israel	rider@ee.technion.ac.il

Name	Affiliation	email
Sniady Piotr	Instytut Matematyczny, Uniwersytet Wrocławski, Poland	`Piotr.Sniady@math.uni.wroc.pl`
Speicher Roland	Queen's University Jeffery Hall, Canada	`speicher@mast.queensu.ca`
Spiridonov Vyacheslav	Joint Institute for Nuclear Research; Dubna, Russia	`svp@thsun1.jinr.ru`
Spitz Grigory	International Sophus Lie Centre, Moscow, Russia	
Starr Shannon	U.C. Davis, USA	`sstarr@math.ucdavis.edu`
Stenlund Mikko	Department of Mathematics, University of Helsinki, Finland	`mikko.stenlund@helsinki.fi`
Stevens Laura	UNC Chapel Hill, USA	`laura@jalapeno.math.unc.edu`
Strahov Evgeny	Brunel University, UK	`strahov@physics.technion.ac.il`
Stukopin Vladimir	Don State Technical University, Russia	`stukopin@math.rsu.ru`
Suidan Toufic	Princeton University; USA	`tmsuidan@math.princeton.edu`
Szenes Andras	MIT, USA	`szenes@math.mit.edu`
Tarasov Vitaly	Steklov Institute of Mathematics at St. Petersburg (POMI), Russia	`tarasov@pdmi.ras.ru`
Terilla John	UNC, USA	`terillaj@wlu.edu`
Tsilevich Natalia	St. Petersburg State University, Russia	`natalia@pdmi.ras.ru`
Vasiliev Vladimir	VNIPIET, Russia	
Vershik Anatoly	Steklov Institute of Mathematics at St. Petersburg (POMI), Russia	`Vershik@pdmi.ras.ru`
Wojakowski Lukazc	University of Wroclaw, Mathematical Institute, Poland	`Lukasz.Wojakowski@math.uni.wroc.pl`
Yakubovich Yuri	Steklov Institute of Mathematics at St. Petersburg (POMI), Russia	`yuyakub@mail.ru`
Yambartsev Anatoly	Moscow State University, Russia	`yambar@lbss.math.msu.su`
Yakobson Dmitri		
Zamyatin Andrej	Lab. of Large Random Systems, Moscow State Univ., Russia.	`zamyatin@lbss.math.msu.su`
Zinn-Justin Paul	LPTMS, University Paris-Sud, ORSAY, FRANCE	`Paul.Zinn-Justin@lpt.ens.fr`
Zvonarev Mikhail	Steklov Institute of Mathematics at St. Petersburg (POMI), Russia	`zvonarev@pdmi.ras.ru`
Zvonkine Dimitri	Ecole Normale Superieure, France	`zvonkine@clipper.ens.fr`