Robert Adol’fovich Minlos

Robert Adol'fovich Minlos (1931–2018) His Work and Legacy

The renowned mathematician Professor Robert Adol'fovich Minlos passed away on 9 January 2018, at the age of 86. An eminent researcher and outstanding teacher, he was a world-renowned specialist in the area of functional analysis, probability theory, and contemporary mathematical physics.

R. A. Minlos was born on 28 February 1931 into a family with a strong connection to the humanities. His father, Adol'f Davidovich Miller, was known as a lecturer and was the author of English dictionaries and manuals. His mother, Nora Romanovna (Robertovna) Minlos, was a historian-ethnographer. This is perhaps why Robert Adol'fovich loved poetry, wrote verses himself, was a fervent theater-goer from his school years, and was began painting seriously at the age of 40.

Nothing foreshadowed a mathematical future, but when he was 15, the young Robert accidentally saw a poster about the Moscow Mathematical Olympiad for schoolchildren. He participated in it, obtained the second prize and, inspired by that, began to attend the school club led by E. B. Dynkin. In 1949 Robert already entered the Faculty of Mechanics and Mathematics of the Moscow State University. He continued to participate in Dynkin's seminar, which together with A. S. Kronrod's seminar, had a great influence on him as an undergraduate student.

R. A. Minlos prepared his first scientific paper (equivalent to a master's degree thesis) in 1950 while participating in the Moscow State University seminar on the theory of

Still life with flowers. R. A. Minlos.

functions of a real variable under the leadership of A. S. Kronrod. But the real scientific interests of the young mathematics student began to form after he became acquainted with I. M. Gelfand. Their joint publication "Solution of the equations of quantum fields" (Doklady Akad. Nauk SSSR, n.s., 97, 209–212, 1954) became Minlos' Diploma thesis in mathematics. It was devoted to the *functional*, or, in mathematical physics language, the *path* integral, which has a direct relation to quantum physics.

As Minlos himself admitted: "My further life in mathematics was predetermined by that work, because I was subsequently mainly occupied with mathematical physics. There were, nevertheless, more works on random processes, on measure theory, and on functional analysis." Very soon one of his papers "Extension of a generalised random process to a completely additive measure" (Doklady Akad. Nauk SSSR, 119, 439–442, 1958) brought Minlos worldwide fame. It became the basis of his Candidate (equivalent to PhD) dissertation "Generalised random processes and their extension to a measure", which was published in Trudy MMO, 8, 497–518, 1959. This result, which is important for the theory of random processes as well as for functional analysis, is now known as the *Minlos theorem* on the extension of cylindrical measures to Radon measures on the continuous dual of a nuclear space, i. e. the continuation of a process to a measure on spaces adjoint to nuclear spaces.

The connection of Minlos to mathematical physics at that time was manifested by the publication (jointly with I. M. Gelfand and Z. Ya. Shapiro) of the monograph "Representations of the rotation and Lorentz groups and their applications" (1958), which was later translated from the Russian by Pergamon, London, in 1964.

From 1956 to 1992, R. A. Minlos was employed by the Department of the Theory of Functions and Functional Analysis of the Faculty of Mechanics and Mathematics at the Moscow State University (MSU). In that period, there was a need to organise a joint seminar with F. A. Berezin, primarily to discuss the mathematical problems of quantum mechanics and of quantum field theory.

A real advance of activity in the field of mathematical physics at the Faculty of Mechanics and Mathematics of MSU was achieved with R. A. Minlos and R. L. Dobrushin's organisation of a seminar on statistical physics in 1962. It soon became widely known in the Soviet Union and abroad as the *Dobrushin-Malyshev-Minlos-Sinai* seminar. The quantum aspects of statistical mechanics at the seminar were primarily associated with the name of R. A. Minlos. The seminar lasted until 1994 and had a huge impact on the

R. A. Minlos with H. Zessin and S. Poghosyan, Bielefeld 2005.

world of modern mathematical physics. Almost all celebrated specialists in this field visited Moscow during the lifespan of the seminar.

The beginning of the 1960s was extremely fruitful for Robert Adol'fovich. In the first place, this concerns new results obtained jointly with L. D. Faddeev on the quantum mechanical description of three particles (1961). It was followed by two articles devoted to study of the thermodynamic limit in classical statistical physics (1967). There R. A. Minlos suggested the first rigorous mathematical definition of the limiting Gibbs distributions for an infinite system of interacting classical particles and also analysed the properties of such distributions (Funct. Anal. Appl., 1, 140–150 and 206–217, 1967). This result anticipated the origin of the Markovian understanding of the Gibbs random fields in the sense of *Dobrushin-Lanford-Ruelle* (1968).

The result (together with Ya. G. Sinai) of the appearance of phase separation in lattice systems at low temperatures (Math. USSR-Sb., 2, 335–395, 1967; Trudy MMO, 17, 213–242, 1967 and 19, 113–178, 1968) was of fundamental importance for the mathematical theory of phase transitions. It formed the basis of Minlos' doctoral dissertation, which he submitted for habilitation in 1968. In another joint work with Ya. G. Sinai (Theor. Math. Phys., 2, 167–176, 1970) the foundation was laid for a new approach to the study of the spectral properties of many-particle systems. In combination with the cluster expansions, this approach drove significant progress in the description of the properties of such infinite systems, including the spectrum of elementary particles of quantum fields and the mathematical description of the *quasi-particle* picture in statistical physics.

R. A. Minlos with participants of the conference, Tsaghkadzor 2002.

The new powerful method of cluster expansions was, from the very beginning, central in the list of interests of Robert Adol'fovich. He proposed to develop further the cluster expansion method so that it could be applied to the study of what he called cluster functions.

These are correlation functions, truncated correlation functions, semi-invariants and the Ursell functions. It was his idea to use the so-called Ursell kernel for the representation of the cluster functions. In his paper with S. Poghosyan (1977) a fundamental forest graph estimate of the Ursell kernel, which contains as a special case the well-known tree graph estimate, was obtained. The forest graph estimate was used later by S. Poghosyan and D. Ueltschi to develop an abstract cluster expansion method. Similarly, S. Poghosyan and H. Zessin developed the large volume geometric expansion of the log-partition function for the models of classical and quantum physics and for construction of the Gibbs processes.

The results of a large series of papers on cluster expansions by R. A. Minlos, V. A. Malyshev, and by their students have been summarised in two monographs "Gibbs random fields: Cluster expansions" (Springer 1991, translation of 1985 Russian edition) and "Linear infinite-particle operators" (Amer. Math. Soc. 1995, translation from the Russian edition of 1994). As was outlined in the book "Gibbs random fields", the method of cluster expansions provides not only a construction of the limiting Gibbs measure but

R. A. Minlos with participants during the excursion, Tsaghkadzor 2002.

also cluster representations of the projections of the limiting Gibbs measure onto bounded regions.

A famous peculiarity of the Dobrushin-Malyshev-Minlos-Sinai seminar was not only its duration of about four hours, which was amazing for foreign guests, or the assertive directness in communicating with lecturers, but also the opportunity to obtain from the discussions some interesting problems to be solved. In essence, the seminar was functioning as a *machine*, generating questions and a possible way to convert them into answers. Robert Adol'fovich was always one of the sources of interesting questions and open problems. The list of projects thus originated includes, for example, the cluster expansions and their applications to the problem of uniqueness/non-uniqueness of the Gibbs states, the quantum three-particle problem, the Trotter product formula for Gibbs semigroups, the study of infinite-particle operators spectra, the analysis of quasi-particle picture in statistical physics, and many others.

In their book "Linear infinite-particle operators" V. A. Malyshev and R. A. Minlos proposed a description of a quasi-particle picture based on the construction by cluster expansions of the lower branches of the spectrum of infinite many-body system with a good clustering. This idea goes back to the paper by R. A. Minlos and Ya. G. Sinai "Investigation of the spectra of some stochastic operators arising in the lattice gas models" (1970). If the system possesses a good clustering one can construct separated translation-invariant two-, three-, and more (interacting) quasi-particles excited states, which are combinations of branches with bands of continuum spectra. Robert Adol'fovich called this property of excitations "The *corpuscular* structure of the spectra of operators describing large systems" (title of his paper in *Mathematical Physics* 2002, Imperial Coll. Press 2000).

In addition to the Dobrushin-Malyshev-Minlos-Sinai seminar in the 1970s, there was a regular tutorial seminar, which was led by Robert Adol'fovich once a week. This was a very good opportunity to learn elements of topological vector spaces, in particular the Minlos theorem about the extension of a generalized random process to a measure on spaces adjoint to nuclear spaces. The seminar also covered elements of mathematical statistical physics in the spirit of the famous "Lectures on statistical physics" in Uspekhi Math. Nauk (1968).

R. A. Minlos as well as Ya. G. Sinai and R. L. Dobrushin were often invited by the Institute of Mathematics in Yerevan to participate in regular conferences under the name

R. A. Minlos with participants of the conference, Lake Sevan 2006.

"Probabilistic methods in modern statistical physics". The first one was held in 1982 and the last one in 1988, three years before the collapse of the Soviet Union.

The conferences restarted in 1995 at the international level. Robert Adol'fovich participated, as a rule with his students, in all of them, including the conference in Lake Sevan in 2006.

In the early nineties Robert Adol'fovich began his collaboration with Italian institutions and mathematicians. He was a guest of the Department of Mathematics at the university of Rome "La Sapienza" many times, and he also visited other institutions in Trieste, Naples, L'Aquila and Camerino.

A first result had been obtained by C. Boldrighini, I. A. Ignatyuk, V. A. Malyshev and A. Pellegrinotti on the annealed model of a discrete-time random walk on a d-dimensional lattice in mutual interaction with a dynamic random environment. Robert Adol'fovich proposed to apply the results that he had obtained, together with V. A. Malyshev and their students, on the spectral analysis of the transfer matrix for perturbed homogeneous random fields.

Robert Adol'fovich was a wonderful teacher, a patient and wise mentor. Directness, accessibility, and enthusiasm attracted numerous students and followers to him. Many of his later PhD students benefited from a direct generous contact with the *Master*. At

R. A. Minlos on the way to Hayravank Monastery, Lake Sevan 2006.

the Faculty of Mechanics and Mathematics at MSU the student seminar was combined with lectures and scientific seminars guided by Robert Adol'fovich first together with F. A. Berezin and then with V. A. Malyshev. The lecture notes gave rise to many nice and popular tutorial books, for example "Introduction to mathematical statistical physics", published by R. A. Minlos in Univ. Lect. Series, vol.19, AMS 2000.

The problems related to the theory of operators and to quantum physics should be especially noted. A long paper ("A system of three quantum particles with point-like interactions", Russian Math. Surveys, 69, 539–564, 2014) was published by R. A. Minlos on this topic. His very last manuscript (with C. Boldrighini, A. Pellegrinotti, and E. A. Zhizhina) was on the subject "Regular and singular continuous time random walk in dynamic random environment".

To his students and collaborators, Robert Adol'fovich was like a brilliant sculptor who, from a shapeless block, cutting off excess, could create a mathematical masterpiece.

Robert Adol'fovich selflessly served science, and in everyday life he was a generous and friendly person. He gladly shared his enthusiasm and energy with his students and colleagues. In addition to the accuracy of reasoning and complicated techniques, there is always a beautiful idea and harmony in his works. To the question "What three math-

ematical formulas are the most beautiful?" Robert Adol'fovich gave the answer: "The Gibbs formula, the Feynman-Kac formula and the Stirling formula."

Always surrounded by relatives and loved ones, and also by loving pupils, colleagues, and friends, Robert Adol'fovich Minlos lived a complete life. In each of those who knew Robert Adol'fovich, he left a bright drop of memory of himself.

C. Boldrighini (Istituto Nazionale di Alta Matematica, Unità locale Università Roma Tre),
V. A. Malyshev (Faculty of Mechanics and Mathematics, Lomonossov MSU),
A. Pellegrinotti (Dipartimento di Matematica e Fisica, Università Roma Tre),
S. K. Poghosyan (Institute of Mathematics of the NAS RA, Yerevan),
Ya. G. Sinai (Department of Mathematics, Princeton University),
V. A. Zagrebnov (Institut de Mathématiques de Marseille),
E. A. Zhizhina (Institute for Information Transmission Problems, Moscow)

A contribution to the conference in commemoration of R. A. Minlos

> *Science and all of us, we have lost an outstanding mathematician. I have lost my best friend and best co-author. But most important, Robert was a reliable friend, kind and human in all life aspects. Nowadays in science (as anywhere else) struggle for power, grants, academic positions, often kills science itself. This I never saw in Robert. I often see now our common past with Robert and this provides me strength to struggle for real science against politicians in science. I feel that he sees all of us from another world. The world that we still even do not try to understand because publishing papers for impact factors takes too much time and energy from us. I am sure he feels strong in that world and is ready to help all of us.*
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> *Vadim Malyshev*
> *Moscow, September 2019*

Robert Adol'fovich Minlos